SPIROGRAPH

Spirograph is a geometric drawing device that produces various mathematical curves known as hypotrochoids and epitrochoids.

The well-known toy version was developed by British engineer [Denys Fisher](#) and first sold in 1965.

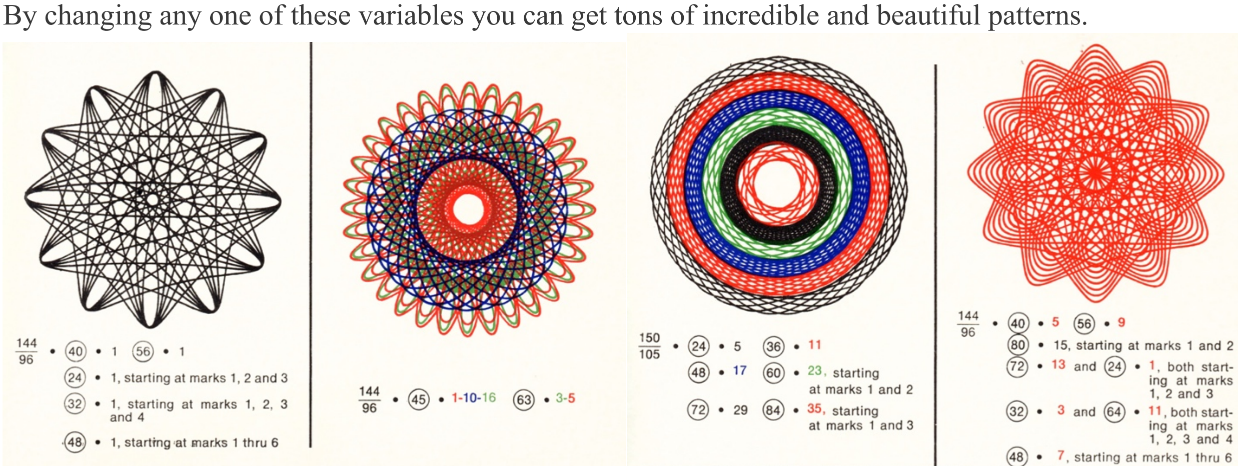
Original toy's website: https://www.kahootztoys.com/spirograph-home.html

This image is taken from [Smithsonian National Museum of American History](#)

The patterns that are created depend on three variables:

- *the radius of the fixed disc or wheel, (the number of teeth)*
- *the radius of the revolving disc, (the number of teeth)*
- *the location of the pen (point) on the moving disc.*

By changing any one of these variables you can get tons of incredible and beautiful patterns.



This images are taken from http://blog.presentandcorrect.com/scans-from-how-to-draw-with-spirograph-1967

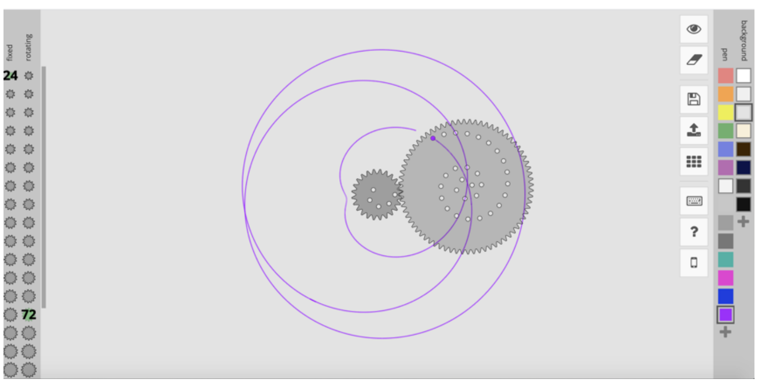
You can check the [Wolfram's collection of plane curves] to identify and see the formulas behind the designs you create !

A point on a wheel rolling inside a circle traces out a hypocycloid.
A point on a wheel rolling on a flat surface traces out a curve called a cycloid.
A point on a wheel rolling outside another wheel traces out an epicycloid.

A spirograph can be used to create artistically interesting patterns. Besides the serious math behind it, the patterns can also be used to study;

- *LCM*
- *clock (modular) arithmetic*
- *the fundamental theorem of mathematics.*

HOW CAN YOU PREDICT HOW MANY POINTS (PETALS) A SPIROGRAPH PATTERN WILL HAVE?

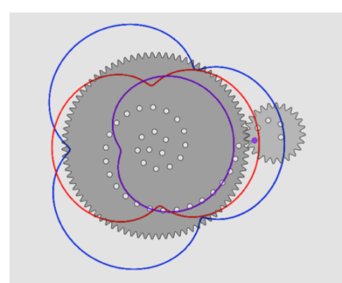


Use the spirograph applet here "https://nathanfriend.io/inspirograph/"

There are two settings to play with about the number of teeth of fixed and the rotating circle on the screen. If you also look at the very bottom of the left side toolbar, you'll also see the wheel & disk version with the 144:96 and 150:105 ratios.

SECTION A:

Part 1: Start with 24:24 for both the fixed and rotating.
Try all the points on the rotating circle. What did you observe?



- Select 48:24 (48 for the fixed)

- In how many loops did the curve close?
- Does the curve always get close?

- Now Select 60:30 – What did you realize?
  Any resemblance with 48:24 – Try the other 2:1 ratios

Part 2: Now try the below in order by leaving the rotating circle with 24 tooth as your constant.

- 24:24
- 36:24
- 48:24
- 60:24
- 72:24 ..

- What do you realize the number of petals? How do they change?
- Do you have a method to generalize it?
- How many trips should the rotating disk take around to complete the cycle?

Hint: You may need a table to organize your findings.

Part 3: Choose 30 and 45

- What happens if 30-tooth disc revolves around 45?
- What happens if 45-tooth disc revolves around 30?

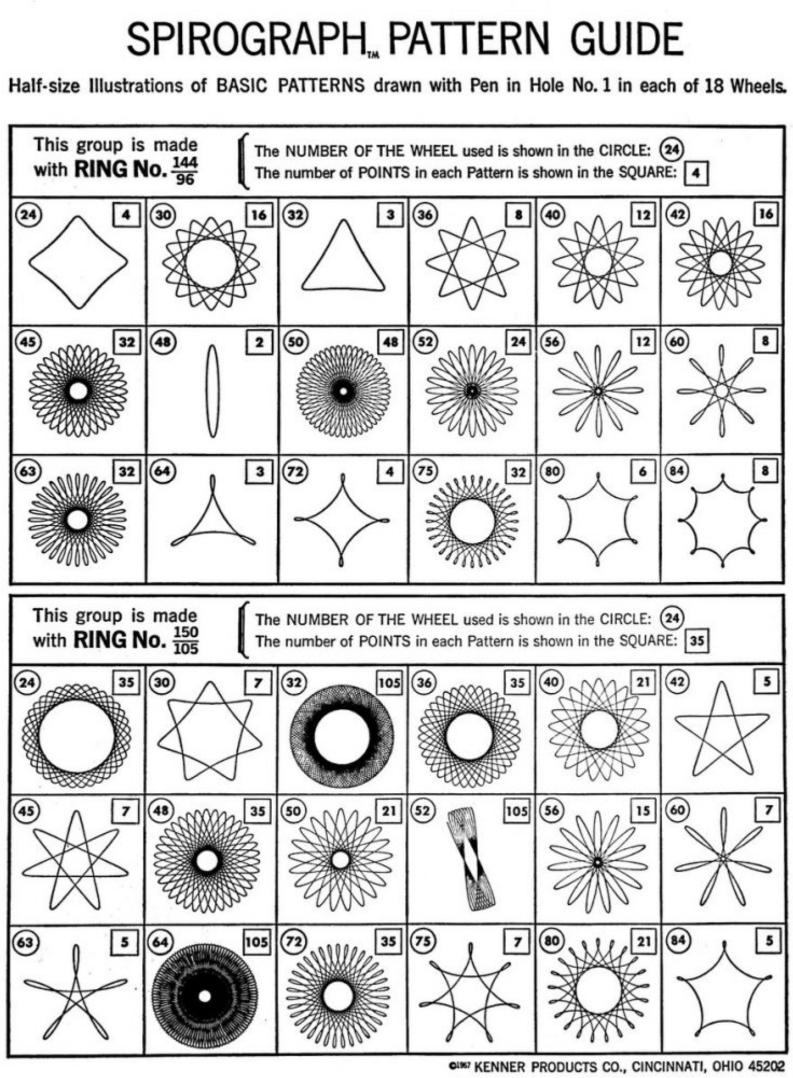
Try with the different numbers

Part 4: Fill in the blanks.

- With a 45 tooth circle and 10 tooth wheel the least common multiple is ….. So to go ….. teeth the inner disk must make ….. complete revolutions of the 45 tooth gear so the pattern repeats after …. revolutions.
- With 42 teeth the least common multiple of 10 and 42 is …… The inner wheel must make….. revolutions before the pattern repeats.
- How many revolutions will be made with gears of 10 teeth and 23 teeth? ……
  The inner disk must make …… revolutions before the pattern repeats.
- What about 60 and 75? The LCM is 300, so the pattern should have ……….loops and it should take ………. trips around to complete the cycle.

Part 5: Select 144:96 wheel to try with 24, 30, 32 .. discs

Select 150:105 wheel to try with 24, 30, 32 .. discs



This image is taken from https://clickamericana.com/toys-and-games/see-how-vintage-spirograph-toys-made-it-easy-for-anyone-to-draw-amazing-geometric-designs

By looking at the data above;

- Write a formula for the N (The number of points/petals) by using the R (the number of teeth on the rotating disc) and W (the number of teeth on the wheel)

N =

- For the wheel 150/105, How many tooth discs could be used to have a shape with 7 points (petals) ?

- For the wheel 144/96, How many tooth discs could be used to have a shape with 8 points (petals) ?

- True or False?

$$\mathrm{N} = \frac{\boldsymbol{LCM\ (R, W)}}{\boldsymbol{W}}$$

N: The number of points/petals
R: the number of teeth on the rotating disc
W: the number of teeth on the wheel

<u>SECTION B:</u>

Can Modular arithmetic also help us to calculate the number of revolutions of the spirograph that are needed to close the pattern?

<u>Part 1:</u> Select the 72:36 ratio

- How many revolutions do you observe?

Select the 72:24 ratio

- How many revolutions do you observe?
- Do you think the number of revolutions is related with the quotient of the R and W?

- 72/36 = 2 no remainder like 72/24 =3 No remainder means that the marked tooth will return to its starting position after …………………..

Part 2: Select the 84:36 ratio

- How many revolutions do you observe?

Let's see what happens …

if there were 84 teeth in the outer ring then 84/36 = 2 remainder 12. The marked tooth misses returning to its original position by 24 teeth!

$84 \equiv 12 \pmod{36}$

One its second time around it rolls around 2 x 84 = 168 teeth and 168/36 = 4 remainder 24. It still misses

$168 \equiv 24 \pmod{36}$

On the third time around it rolls around 3 x 84 teeth = 252 teeth for 252/36 = 7 no remainder and the pattern repeats after 3 revolutions.

$252 \equiv 0 \pmod{36}$

In the spirograph, when the modulus is ……………. the pattern repeats exactly.

- Try with the numbers if your generalization holds!
  - Select 75: 30
  - Select 75: 45

Calculate the number of revolutions with the modular arithmetic and then use the applet to see if your calculation is correct.

RESOURCES
Math behind spirographs
http://mathworld.wolfram.com/topics/Roulettes.html
https://www.tec-science.com/mechanical-power-transmission/cycloidal-gear/geometry-of-cycloidal-gears/
https://archive.bridgesmathart.org/2009/bridges2009-279.pdf

GeoGebra applets:
http://www.malinc.se/math/trigonometry/spirographen.php

Research:
https://en.wikipedia.org/wiki/Spirograph
https://www.sciencekiddo.com/spirograph-math/
http://www.exo.net/~pauld/activities/spirograph/Spirograph.html
http://spirographicart.com/table-spirograph-points/
http://spirographicart.com/spirograph-pattern-guide/

If you like to create your own Spirograph machine by using lego bricks.
You can check out the YouTube and the websites below;

- https://frugalfun4boys.com/build-lego-design-drawing-machine/
- https://www.instructables.com/id/Lego-Spirographer-V10/
- http://spirographicart.com/2016/02/09/lego-spirograph-machine/
- https://www.youtube.com/watch?v=sdWOgE5oqX8