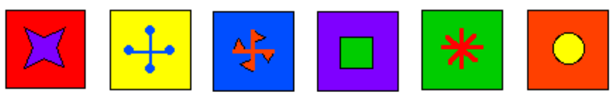
A CLOSER LOOK TO CUBE

PART I: FACES OF A CUBE

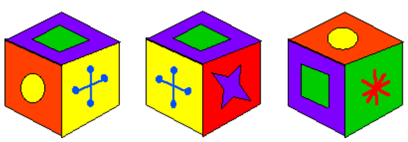
A. A Puzzling Cube from NRICH

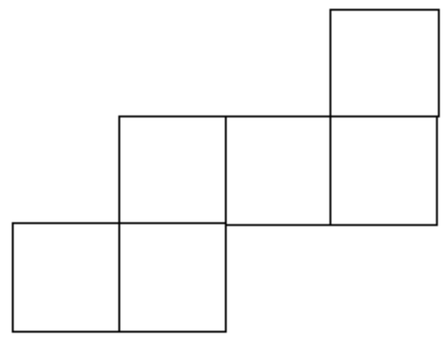
https://nrich.maths.org/1140

Here are the six faces of a cube - in no particular order:



Here are three views of the cube:



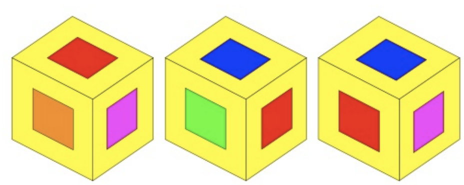


Can you deduce where the faces are in relation to each other and record them on the net of this cube? You can use this interactivity to try out your ideas. You will still have to visualize the cube folded up!

B. DICEY from NRICH

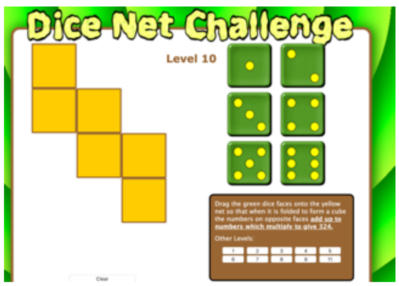
https://nrich.maths.org/993

A game has a special dice with a color patch on each face. These three pictures show different views of the same dice. What color is opposite Blue?



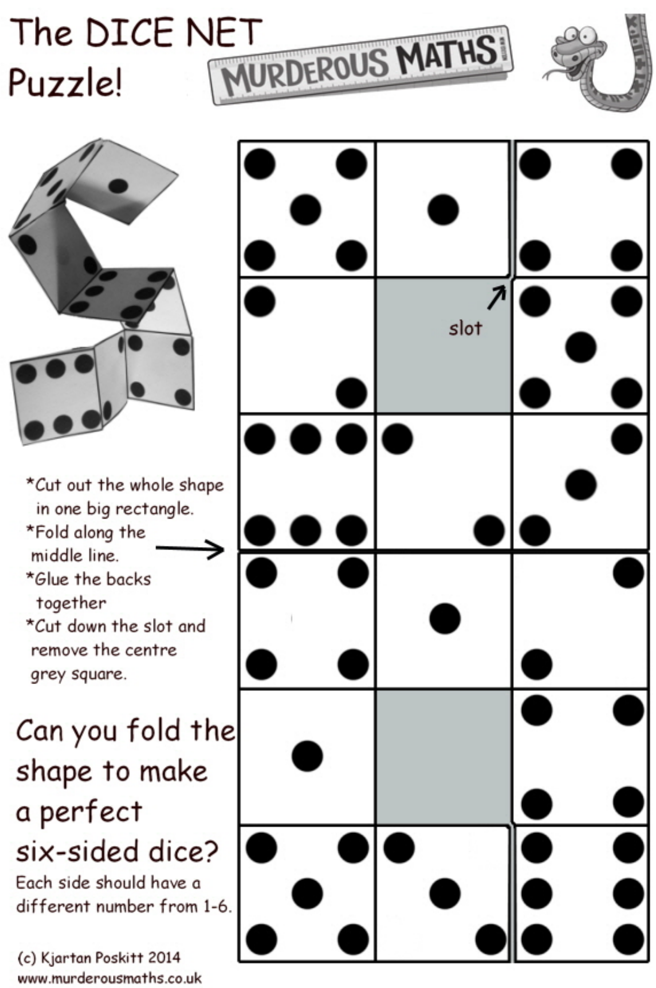
PART II. OPPOSITE FACES OF A DIE

1. What is the sum of the opposite faces of a fair die?



2. By using the website below, use the different nets of a cube to create the fair dice?
Link:

3. Print out and ready to cut the shape given below to see if you can create a fair die?
Link



EXPLORE MORE: How Many Dice?

https://nrich.maths.org/692

Look at a die. Had you noticed before that 1, 2 and 3 are opposite 6, 5 and 4 respectively so that opposite faces add to 7?

I'll call a die which has this property a standard die.

I don't know when this standard convention was adopted, but I know that if you make standard dice by writing 1, 2, 3, 4, 5, 6 on blank cubes you will find there are 2 and only 2 different standard dice.

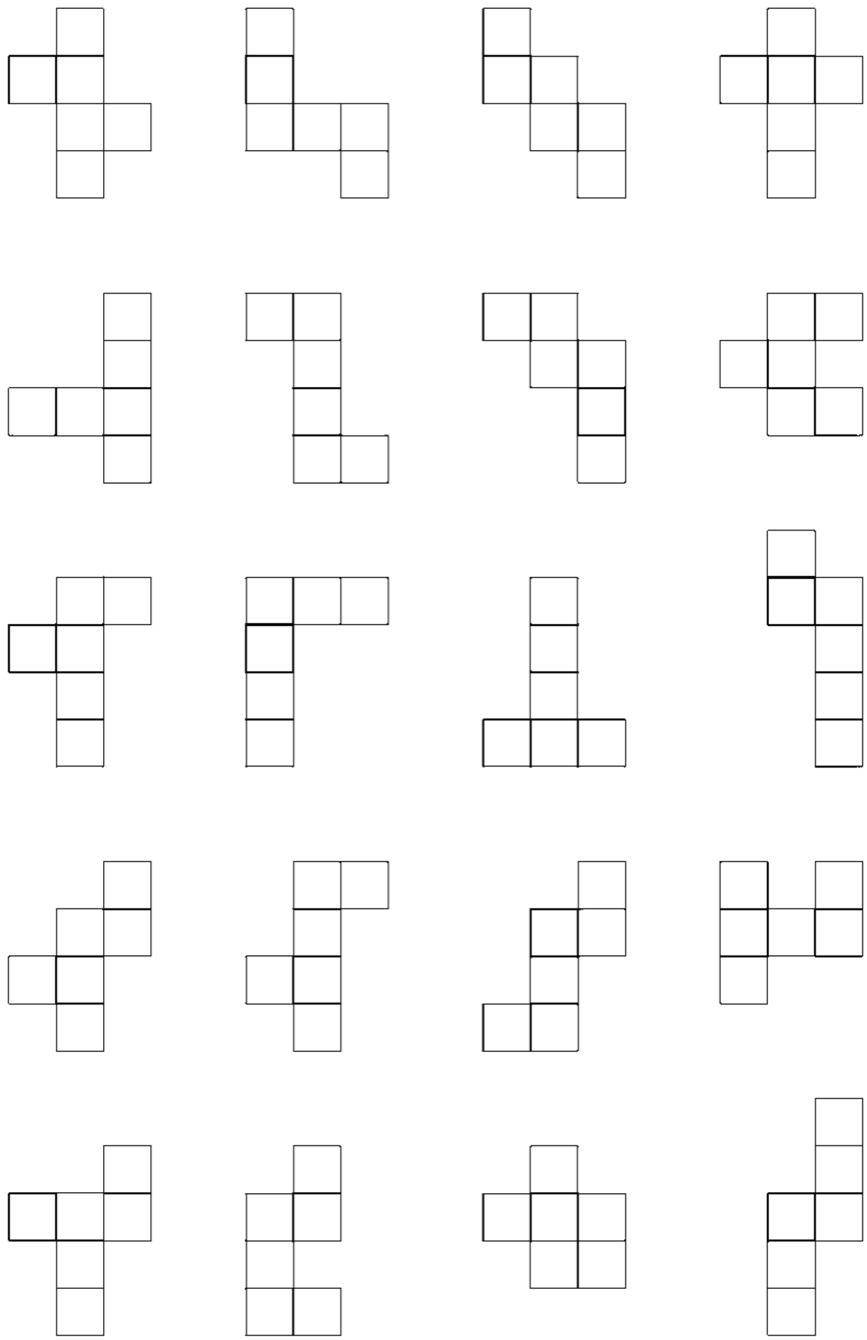
Can you prove this?

A mathematician friend from New Zealand, Prof Derek Holton, tells us that one of the standard dice is used in the western world and the other one is used in the east. Can anyone confirm this with an example?

Suppose we relax the condition that opposite faces sum to 7. How many different dice can we make now?

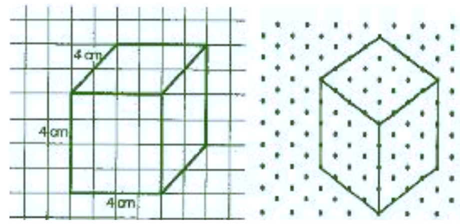
PART III. NETS OF A CUBE

Look at the hexominoes below. Only 11 of them can make a cube when folded. Can you figure out which ones are the nets of the cubes?



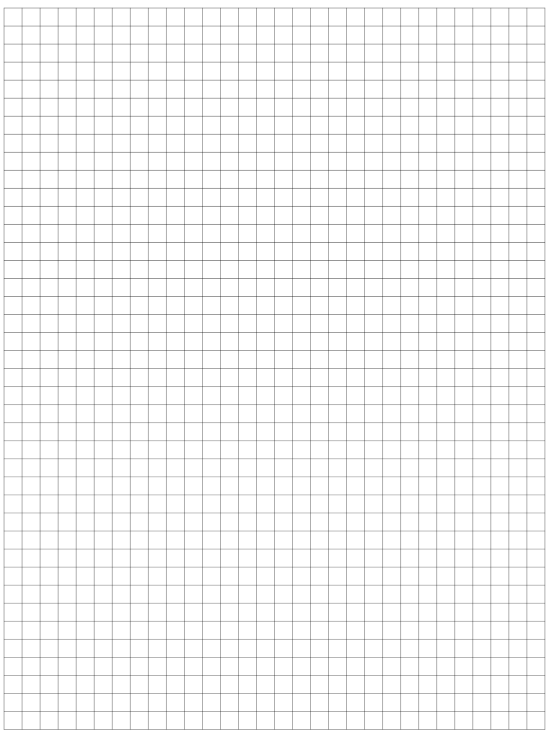
PART IV DRAWING THE CUBE ON DIFFERENT GRIDS

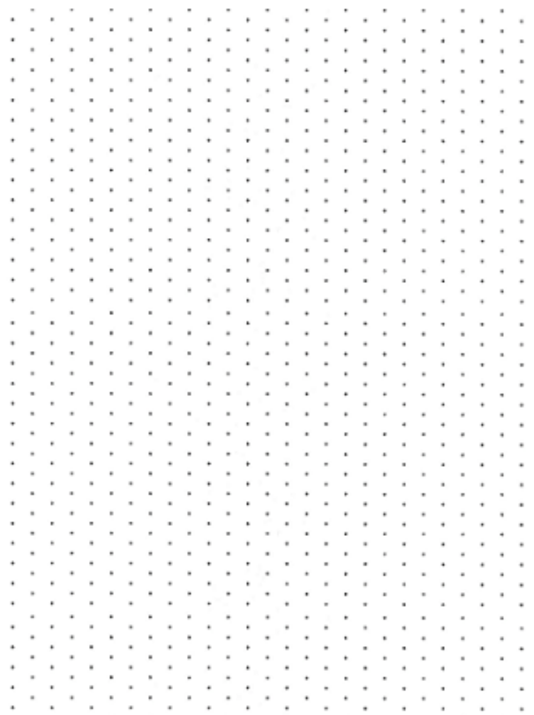
Look at the example of a 4-unit long cube



On both grids;

1. Draw a cube with an edge 6 units long.
2. with an edge 5 units long.
3. Can you come up with an easy way to draw cubes?





 # OF FACES OF A CUBE:

 # OF EDGES OF A CUBE:

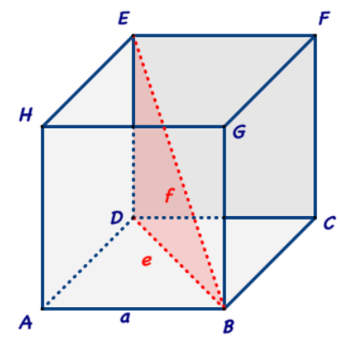
 # OF VERTICES OF A CUBE:

PART V. DIAGONALS

 # OF FACE DIAGONALS OF A CUBE:

 # OF SOLID (SPACE) DIAGONALS OF A CUBE:

1. Use the Pythagorean Theorem to find the length of face diagonal and solid diagonal for a cube. Make a generalization.



For a cube with an edge "a" cm long

Face diagonal = ________ cm whereas the Solid diagonal is __________cm.

2. What are the DIAGONA PLANES of a cube?

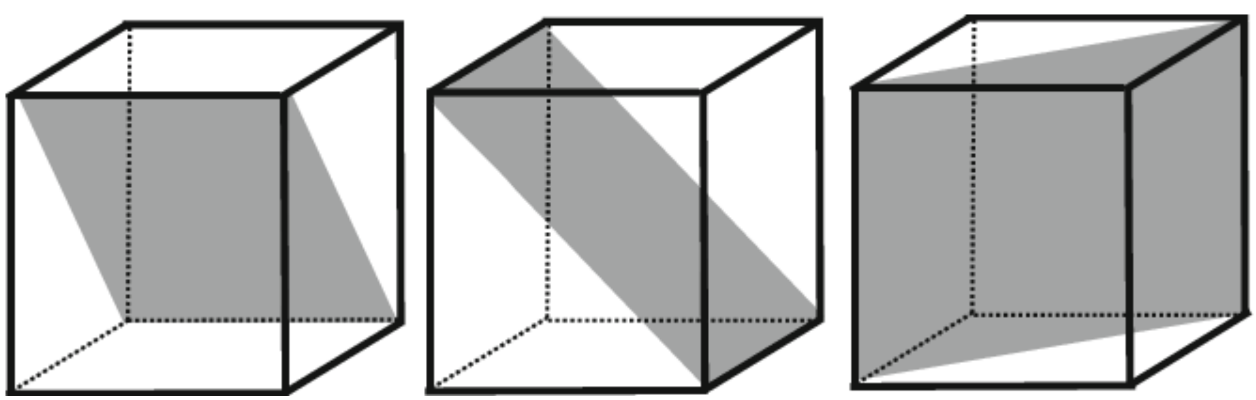
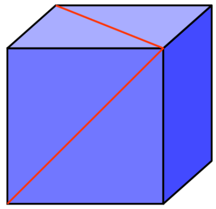


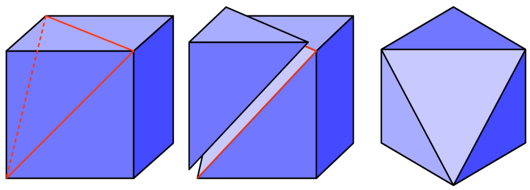
Image is taken from https://www.researchgate.net/figure/Diagonal-planes-that-are-formed-by-cutting-the-cube-diagonally-starting-at-one-of-the_fig1_220490601

Calculate the <u>area</u> of the diagonal planes in terms of the side length of a cube?

3. One of the Martin Gardner's puzzles:
The two red lines in the diagram shown are drawn diagonally on the faces of a cube.
What is the angle between them at the point where they join?



SOLUTION HINTS;



Images are taken from http://puzzles.nigelcoldwell.co.uk/fortyfour.htm

PART VI. SQAURE & CUBE LAW

The **square–cube law** is a mathematical principle, applied in a variety of scientific fields, which describes the relationship between the volume and the surface area as a shape's size increases or decreases. It was first described in 1638 by Galileo Galilei in his *Two New Sciences*.

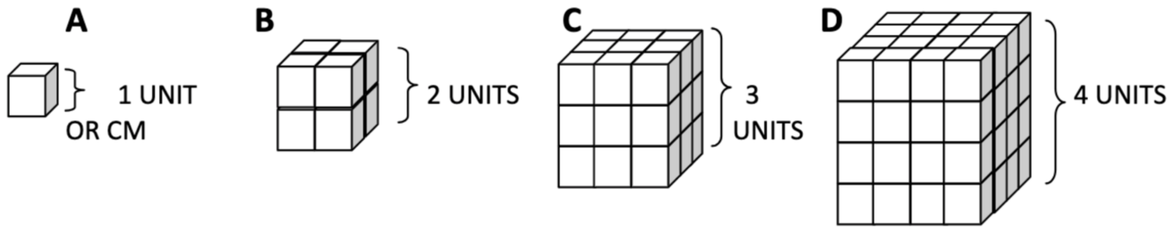
It helps explain phenomena including why large mammals like elephants have a harder time cooling themselves than small ones like mice, and why building taller and taller skyscrapers is increasingly difficult.

1. Read the Applications od square- cube law at
https://www.dinosaurtheory.com/scaling.html

2. Read this to see how **Marvel Studios** use the Square – Cube Law

https://marvel.fandom.com/wiki/Glossary:Square-cube_law

3. Make an AREA – VOLUME CHART for the cubes below;



4. Write your generalizations about how to find the area and volume of a cube and the scale of growth.

5. Look at the wolfram demo above to see the ratio between area and the volume of the cube.
https://demonstrations.wolfram.com/TheRatioOfSurfaceAreaToVolumeForACubeAndASphere/

What did you realize? At which side- length, the numerical value of area and volume would be the same?

PART VII. LET'S INVOLVE SOME MEASUREMENTS

1. How many liters of water needed to fill a cube with an edge 1 cm? What kind of information you need to answer this question?

2. How many liters of water needed to fill a cube with an edge 1 dm?

3. How many liters of water needed to fill a cube with an edge 1 m?

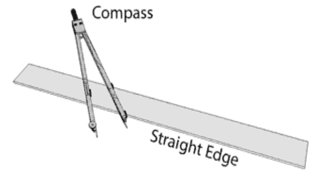
4. What about the mass of those cubes? What kind of information and/or formula you need to answer this question?

5. So, what is the mass of 1m^3 water? (at the temperature of maximum density and standard atmospheric pressure ☺)

6. Can you make an estimation about how many liters of water needed to fill an average pool (and the mass of that amount of water)?

PART VIII. HISTORICALLY FAMOUS "DOUBLING THE CUBE QUESTION"

There are 3 main geometric problems in history (Geometric Problems of Antiquity) whose solutions n was sought using only compass and straightedge:



1. **CIRCLE SQUARING**
2. **CUBE DUPLICATION**
3. **ANGLE TRISECTION**

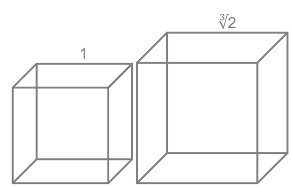
Only in modern times, more than 2000 years after they were formulated, were all three ancient problems proved insoluble using only compass and straightedge.

Doubling the cube, also known as the Delian problem:

Given an edge-length of a cube,

Construct another one whose volume is double that of the first.

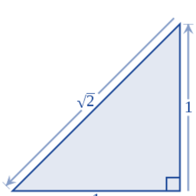
Let's start with a unit cube. Obviously a cube of side length 1 has a volume of $1^3 = 1$,
So our second cube has to have a volume of ____ .
Therefore its one side has to be
________________________________ cm long.



Mathematically speaking;

$x = \sqrt[3]{2}$,

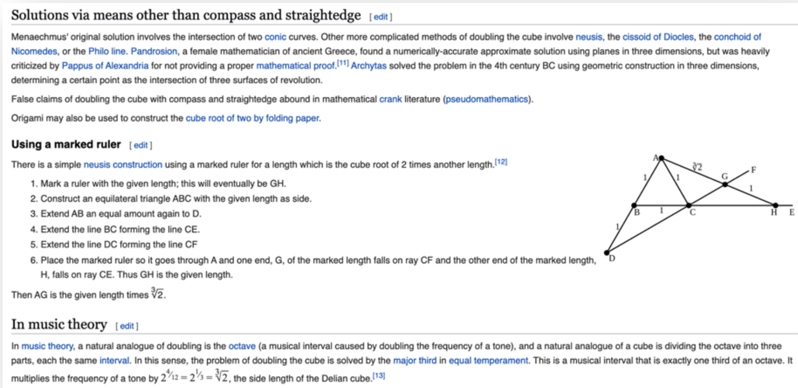
The impossibility of doubling the cube is because $\sqrt[3]{2}$, is not a **constructible number. >>**



The $\sqrt{2}$ is the length of the hypotenuse of a right triangle with legs of length 1 and is therefore, a constructible number.

1. Check the solution of the problem without the restriction of compass and straightedge at

https://en.wikipedia.org/wiki/Doubling_the_cube
or
http://mathworld.wolfram.com/CubeDuplication.html



Solutions via means other than compass and straightedge [edit]

Menaechmus' original solution involves the intersection of two conic curves. Other more complicated methods of doubling the cube involve neusis, the cissoid of Diocles, the conchoid of Nicomedes, or the Philo line. Pandrosion, a female mathematician of ancient Greece, found a numerically-accurate approximate solution using planes in three dimensions, but was heavily criticized by Pappus of Alexandria for not providing a proper mathematical proof.[11] Archytas solved the problem in the 4th century BC using geometric construction in three dimensions, determining a certain point as the intersection of three surfaces of revolution.

False claims of doubling the cube with compass and straightedge abound in mathematical crank literature (pseudomathematics).

Origami may also be used to construct the cube root of two by folding paper.

Using a marked ruler [edit]

There is a simple neusis construction using a marked ruler for a length which is the cube root of 2 times another length.[12]

1. Mark a ruler with the given length; this will eventually be GH.
2. Construct an equilateral triangle ABC with the given length as side.
3. Extend AB an equal amount again to D.
4. Extend the line BC forming the line CE.
5. Extend the line DC forming the line CF
6. Place the marked ruler so it goes through A and one end, G, of the marked length falls on ray CF and the other end of the marked length, H, falls on ray CE. Thus GH is the given length.

Then AG is the given length times $\sqrt[3]{2}$.

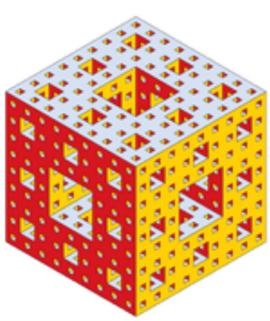
In music theory [edit]

In music theory, a natural analogue of doubling is the octave (a musical interval caused by doubling the frequency of a tone), and a natural analogue of a cube is dividing the octave into three parts, each the same interval. In this sense, the problem of doubling the cube is solved by the major third in equal temperament. This is a musical interval that is exactly one third of an octave. It multiplies the frequency of a tone by $2^{4/12} = 2^{1/3} = \sqrt[3]{2}$, the side length of the Delian cube.[13]

PART IX. VOLUME AND SURFACE AREA OF THE MENGER SPONGE

1. The Menger sponge is constructed by dividing a cube into 27 cubes, then removing the middle cube of each face and the center cube. Think that, this process is repeated at each iteration infinitely many times. By investigating the Wolfram Demo of the Menger sponge, fill in the blanks.

https://demonstrations.wolfram.com/VolumeAndSurfaceAreaOfTheMengerSponge/

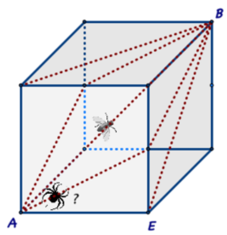


The Demo shows that the surface area of the sponge tends to ______________ for each level of the cube while the volume tends to ________________.

Image is taken from The Guardian. (https://www.theguardian.com/science/2017/apr/10/can-you-solve-it-the-incredible-sponge-puzzle) and suggest you to have a look at the page to see a cooler question "**Menger Slice**" (spoiler alert) about the Menger Sponge.

PART X. SHORTEST DISTANCES ON THE CUBE (Fly vs Spider Problem)

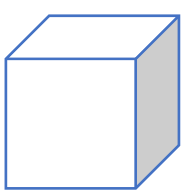
1. Let's say you are a fly at the corner A inside of the cube below who desperately wants to go to the corner B. Since you are one lazy fly, you want to travel the shortest possible distance from A to B. What is that distance? (if being a fly does not interest you assume you have a drone. ☺)
Spider vs fly



2. This time, you are a spider (not spiderman) in the same room, at the same corner and you want to go to the same corner B. What is the shortest possible distance for you? If AE= 4 m, how many meters it takes to travel from A to B ?

3. Given a cube with side length s, and two points A and B on the surface of the cube, how can you find the shortest path along the surface of the cube?

Hint: You may have 3 cases, same face, adjacent faces or opposite faces.
Use the GeoGebra applet https://www.geogebra.org/m/JQ3dTQCm to investigate the different paths.



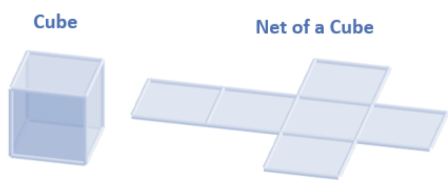
4. *Adobe Interview Question for Developer Program Engineers: (Acc to internet myths)*
There is an ant in a cube placed at one corner and you need to find the shortest path to the diagonally opposite corner. The ant cannot fly!

Guess what Wolfram also has an explanation for that "**fly vs spider problem.**"

http://mathworld.wolfram.com/SpiderandFlyProblem.html

PART XI. WHAT DO LOKI AND S. DALI HAVE IN COMMON?

We know that a 3D cube and its 2D net look likes this;



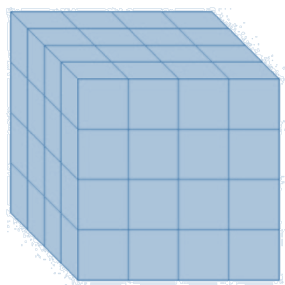
1. Now imagine a 4D Cube and its 3D net. In fact, starting from the net will be easier since it is still 3D. Make your drawings or construct by using unit cubes.

2. How many cubes do you think you will need to construct the 3D net?

3. The 4-dimensional cube is called **tesseract!** I guess you got the Loki connection already. But what about Salvador Dali? Search about how the famous artist Salvador Dali is related with the tesseract.

PART XII: PAINTED CUBE PROBLEM

A solid (4x4) cube of wood is coated with blue paint on all six faces.



1. How many unit cubes are there with 3 painted faces?

2. How many unit cubes are there with 2 painted faces?

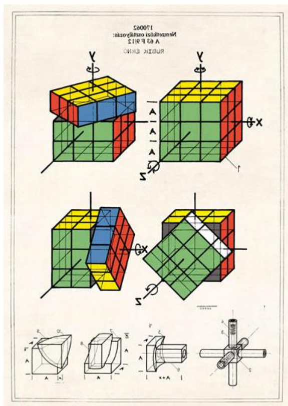
3. How many unit cubes are there with 1 painted face?

4. How many unit cubes are there with 0 painted face?

5. If the initial cube was 5x5, how would those numbers change? (A chart perhaps?)

6. What happens if the cube initially was n x n ?

PART XIII: RUBIKS CUBE



Are you ready to learn some Rubik moves? But first check the first drawings of the cube who born 40 years ago and still one of the best-selling toys of the world?

https://www.rubiks.com/en-us/about

Image is taken from https://www.presentindicative.com/products/rubiks-cube-card

By studying the Rubik's cube movements, you will have an understanding of the axes of symmetry of a cube.

PART XIV: TOYS & APPS

- App suggestion of 3d thinking: Cube Connect Game, iCrosss, Shapes
- For the cubic toys of all kind that help improving your 3d visualization, please check https://www.funmathfan.com/math-gadgets.

- Build your own INFINITY-CUBE.

    - If you have 8 cubes already watch the video for the connection tips; https://www.youtube.com/watch?v=7pEOGPH9y_c
    - Origami infinity cube: https://jonakashima.com.br/2019/07/14/origami-infinity-cube/

PART XV: If you want to explore more about cubes...

- CROSS SECTIONS OF A CUBE
- AXES OF SYMMETRY OF A CUBE
- PLATONIC SOLIDS
- EULER'S POLYHEDRON FORMULA
- SPHERE IN A CUBE
- HYPERBOLOID
- TRISECTING A CUBE
- CUBE ILLUSIONS
- TRUNCATED CUBE
- CUBIC SOAP BUBBLES …