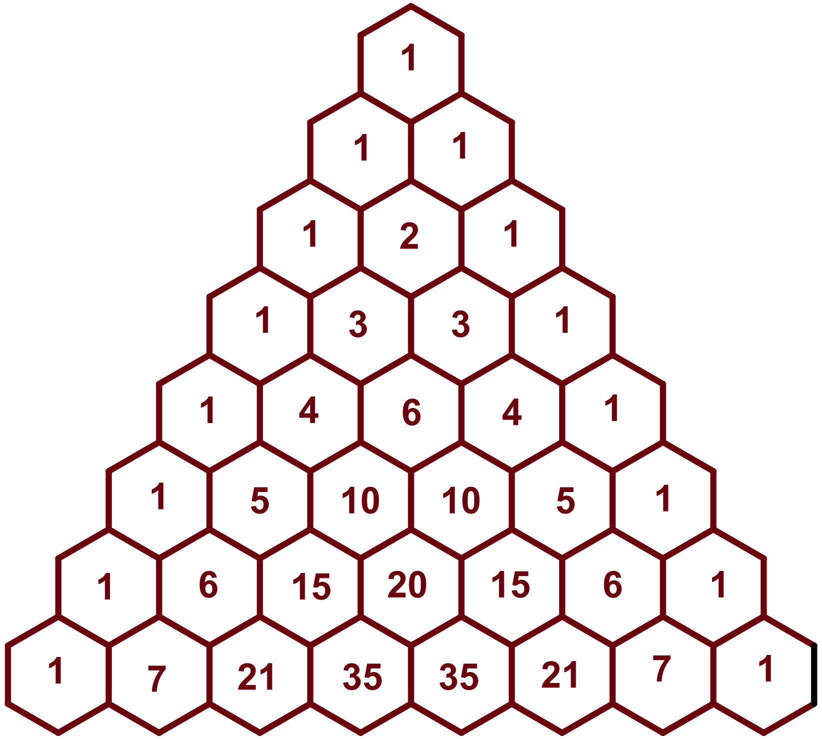
THE MATHEMATICAL SECRETS OF PASCAL TRIANGLE

Pascal's triangle is a never-ending equilateral triangle of numbers that follow a rule of adding the two adjacent numbers above to get the number below. Two of the sides are "all 1's" and because the triangle is infinite, there is no "bottom side."

It is named for Blaise Pascal, a 17thcentury French mathematician who used the triangle in his studies in probability theory. However, it has been studied throughout the world for thousands of years, particularly in ancient India and medieval China, and during the Golden Age of Islam and the Renaissance, which began in Italy before spreading across Europe.

Powers of Two: If we sum each row, we got the powers of 2. //



Sum = 1 = 2^0

Sum = 2 = 2^1

Sum = 4 = 2^2

Sum = 8 = 2^3

Sum = 16 = 2^4

Sum = 32 = 2^5

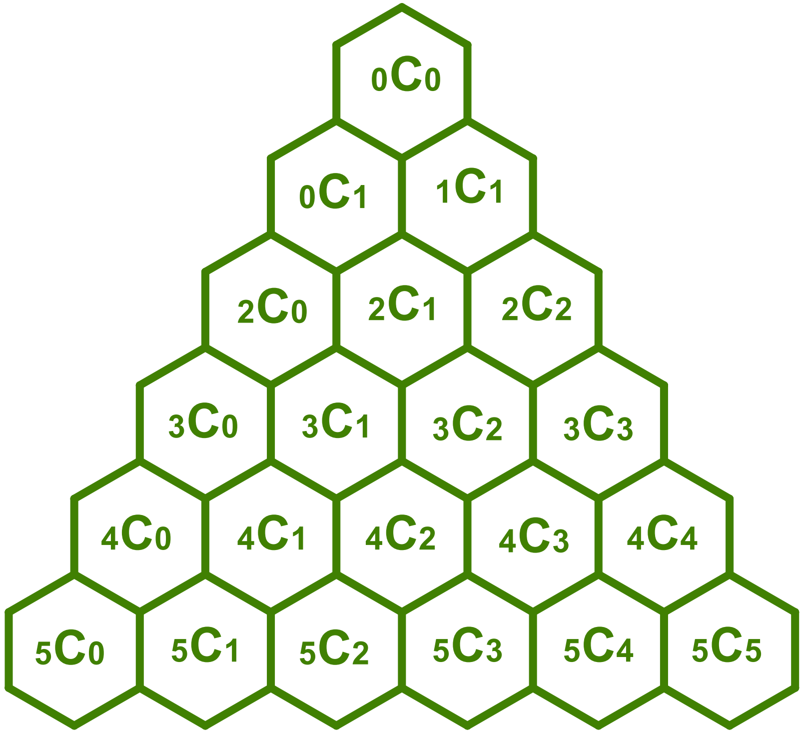
Sum = 64 = 2^6

Sum = 128 = 2^7

✓ Combinations

The numbers of Pascal's triangle match the number of possible combinations ($_nC_r$) when faced with having to choose r-number of objects among n-number of available options.

$$nCr = \frac{n!}{r!\,(n-r)!}$$



✓ The Binomial Theorem

The binomial theorem refers to the pattern of coefficients that appear when a binomial is multiplied by itself a certain number of times.

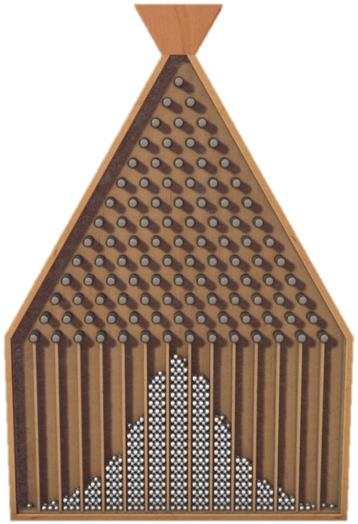
n	$(x + y)^n$	Expanded Polynomial	Pascal's Triangle
0	$(x + y)^0$	1	1
1	$(x + y)^1$	$1x + 1y$	1,1
2	$(x + y)^2$	$1x^2 + 2xy + 1y^2$	1,2,1
3	$(x + y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$	1,3,3,1
4	$(x + y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$	1,4,6,4,1
5	$(x + y)^5$	$1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$	1,5,10,10,5,1
↓	↓	↓	↓

✓ The Binomial Distribution

For a probabilistic experiments with two outcomes (like a coin flip):

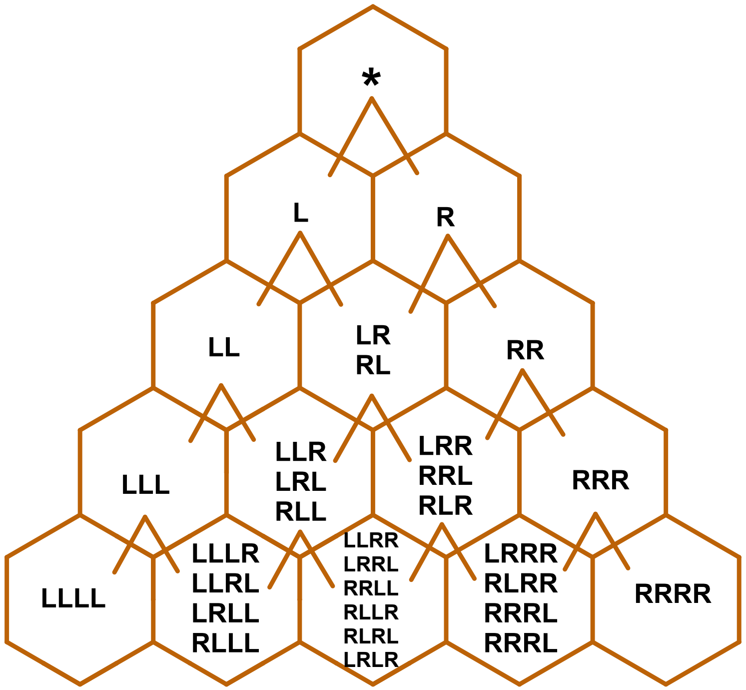
Coin flips	Possible sequences of heads (H) or tails (T)	Pascal's Triangle
1	H T	1 1
2	HH HT TH TT	1 2 1
3	HHH HHT HTH THH HTT THT TTH TTT	1 3 3 1
4	HHHH HHHT HHTH HTHH THHH HHTT HTHT HTTH THHT THTH TTHH HTTT THTT TTHT TTTH TTTT	1 4 6 4 1

✓ The Normal Distribution



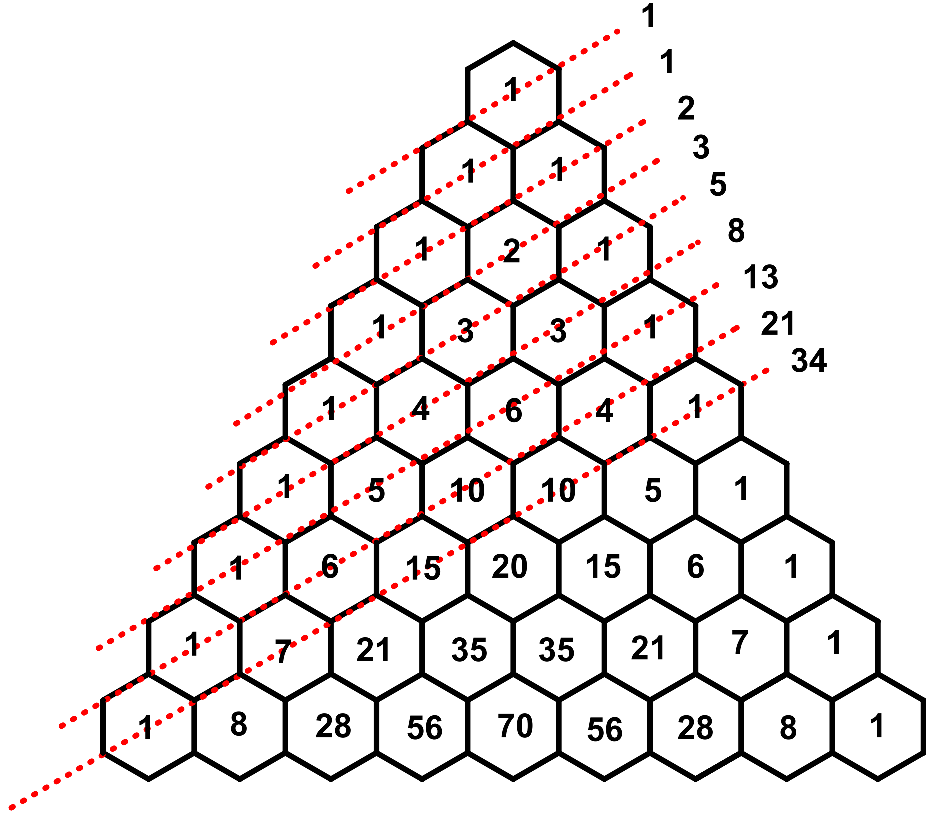
For large numbers of coin flips (above roughly 20), the binomial distribution is a reasonable approximation of the normal distribution, a fundamental "bell-curve" distribution used as a foundation in statistical analysis.

A physical example of this approximation can be seen in a GALTON BOARD ! (A device that randomly sorts balls to bins based on how they fall over a triangular arrangement of pegs.)

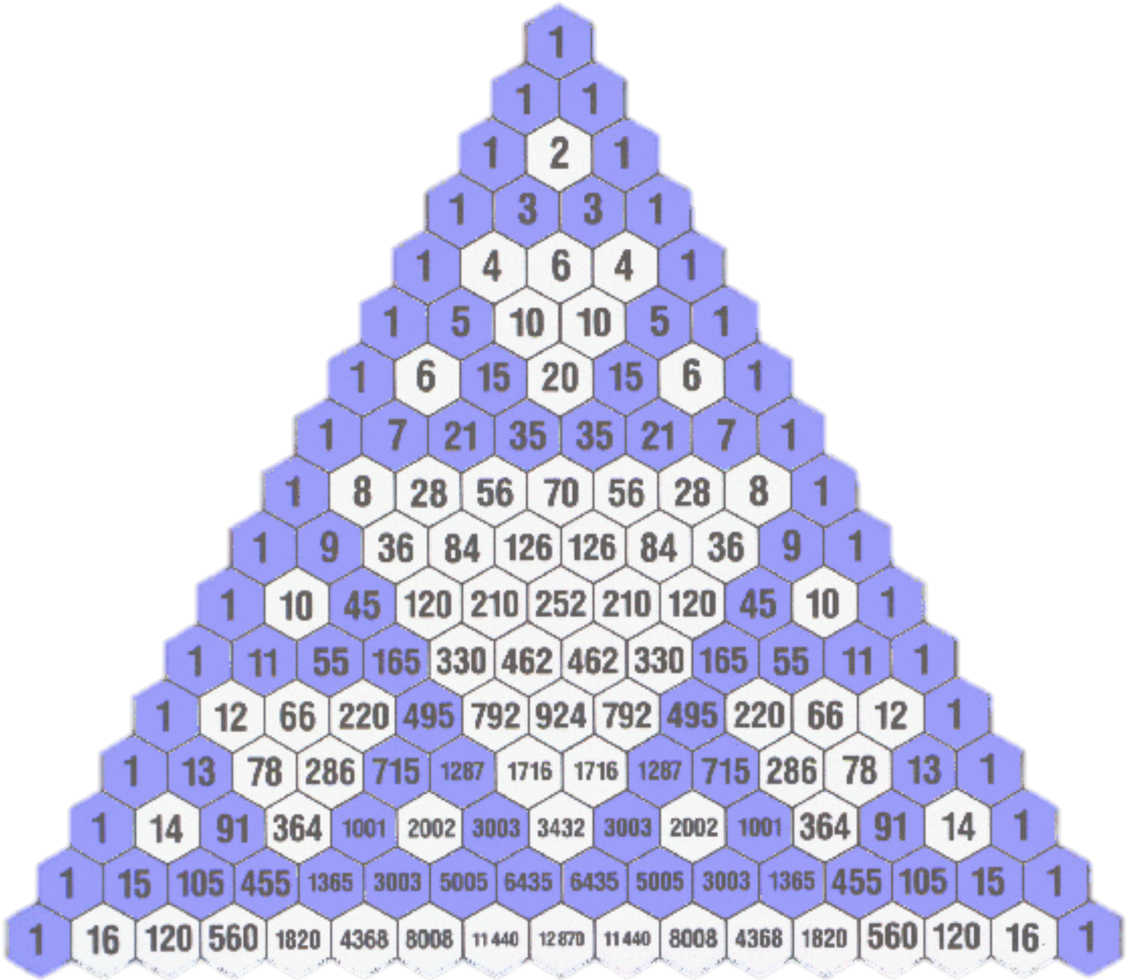


✓ Fibonacci sequence

Adding the numbers of Pascal's triangle along a certain diagonal produces the numbers of the sequence.



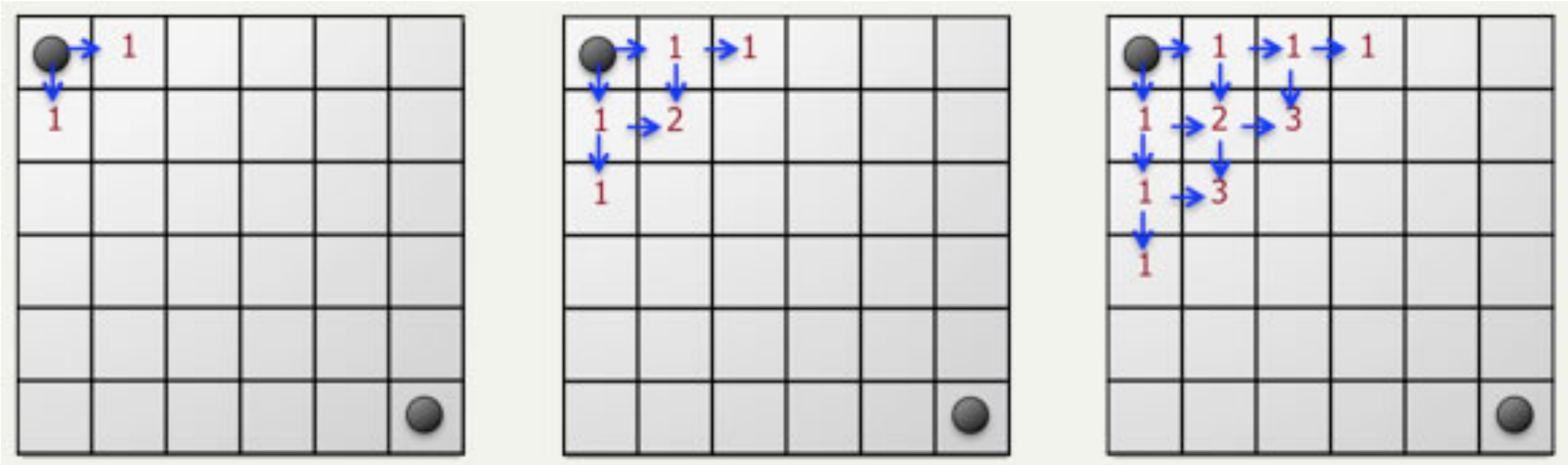
✓ Fractals



Coloring the numbers of Pascal's triangle by their divisibility produces an interesting variety of **fractals**. In particular, coloring all the numbers divisible by two (all the even numbers) produces the **Sierpiński triangle**. //

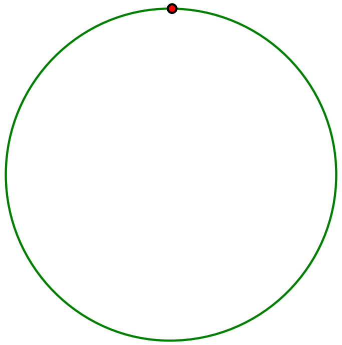
✓ Computational Thinking

You must start in the top left corner and you must end in the bottom right corner. You are only allowed to move down and to the right (no moving up or left). To the right is a 6x6 grid (6 rows and 6 columns). How many paths exist from the top left square, to the bottom right square?

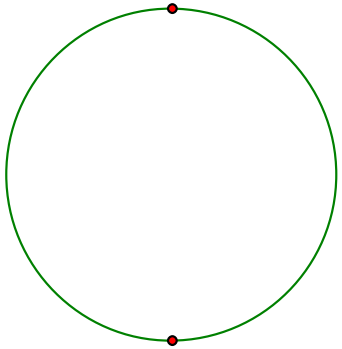


Continue calculating the number of unique paths that lead to each square in the grid.

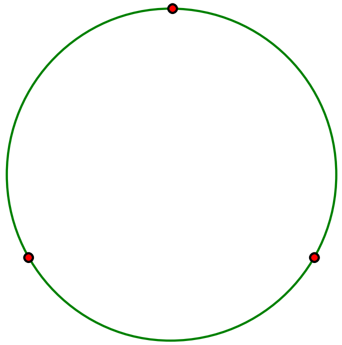
✓ PASCAL TRIANGLE IN CIRCLES



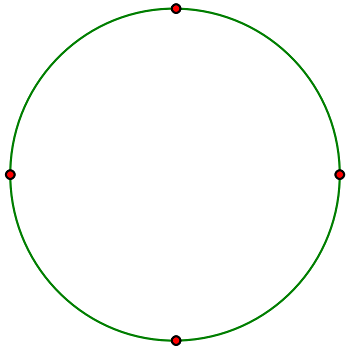
# of circles:
# of points:



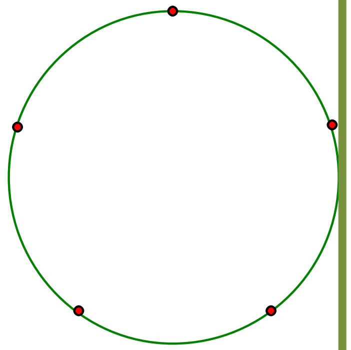
# of circles:
# of points:
# of segments:



# of circles:
# of points:
# of segments:
# of triangles:



# of circles:
# of points:
# of segments:
# of triangles:
# of quads:



# of circles:
# of points:
# of segments:
# of triangles*:
# of quads:
# of pentagons:

* : do not count compound triangles which are formed by two or more triangles.

TASK CARD #1

1. FIND THE POWERS OF 2.
2. FIND THE POWERS OF 11.
3. FIND ALL PERPECT SQUARE NUMBERS (They are hidden well!).
4. FIND FIBONACCI SEQUENCE.
5. FIND THE SIERPINSKI TRIANGLE (Coloring according a pattern can be needed!).

TASK CARD #2

BELOW RELATION EXISTS BETWEEN THE BINOMIAL EXPANSIONS AND THE PASCAL TRIANGLE.

$(x + y)^0$	1
$(x + y)^1$	$1x + 1y$
$(x + y)^2$	$1x^2 + 2xy + 1y^2$
$(x + y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$
$(x + y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$
$(x + y)^5$	$1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$

ACCORDING TO THIS, WRITE

$(x + y)^8 =$

TASK CARD #3

THE **BINOMIAL DISTRIBUTION** DESCRIBES A PROBABILITY DISTRIBUTION BASED ON EXPERIMENTS THAT HAVE TWO POSSIBLE OUTCOMES. THE MOST CLASSIC EXAMPLE OF THIS IS TOSSING A COIN.

Consider flipping a fair coin 3 times. WRITE ITS SAMPLE SPACE.

- Find the probability of getting 3 heads?
- Find the probability of getting 2 heads and a tail?
- Find the probability of getting 2 tails and a head?
- Find the probability of getting 3 tails?

Anything you realize? ☺

Now flip it 4 times.

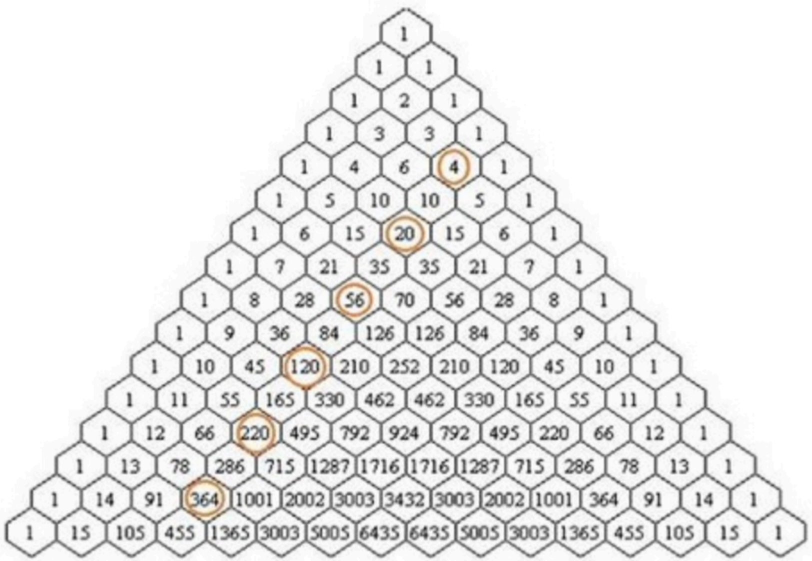
- P (4 HEADS) =
- P (3Hs and 1T) =
- P (2Hs and 2Ts) =
- P (3Ts and 1H) =
- P (4 TAILS) =

ARE YOU READY TO FLIP IT ONE MORE TIME? !!

TASK CARD #4

EVEN YOU CAN FIND π IN PASCAL TRIANGLE

$$\pi = 3 + \frac{2}{3}\left(\frac{1}{C_3^4} - \frac{1}{C_3^6} + \frac{1}{C_3^8} - \cdot\right).$$



BY USING A CALCULATOR, PROVE THAT PI EXISTS IN THE PASCAL TRIANGLE.

TASK CARD #5

HOW CAN YOU FIND **ZERO** IN EACH ROW?

TASK CARD #6

How many ways are there to choose 2 objects from a set of 4? It doesn't take too long to list w/ these numbers {A, B, C, D}. So this is the answer;

AB, AC, AD, BC, BD, CD.

Say, 3 objects from a set of 5. The set is {A, B, C, D, E} and here are the 10 possible groups of objects (listed in alphabetical order):

ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

So the mathematical term for "the number of ways to choose k objects from a set of n objects", we will simply say "n choose k".

is called COMBINATION.

And can easily be calculated as $C(n,k) = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

BELIEVE IT OR NOT, PASCAL TRIANGLE CAN ALSO HELP YOU TO CALCULATE THE NUMBER OF COMBINATIONS OF CHOOSING K OBJECTS FROM N OBJECTS.

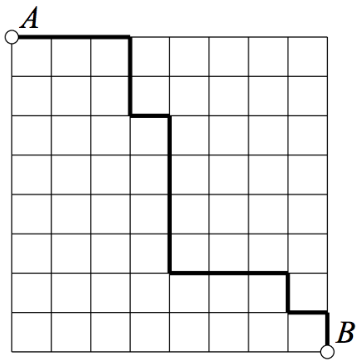
TRY TO FIND OUT HOW!?

TASK CARD #7

PASCAL TRIANGLE CAN EVEN GUESS YOUR EXAM SCORE. EVEN ALL YOUR CLASSMATES'.

Let's draw a diagram for *binomial distribution if each question is answered right (R) or (W)*

TASK CARD #8

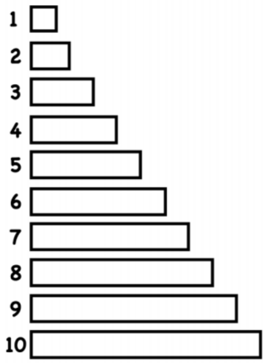


FIND THE NUMBER OF PATHS FROM ONE CORNER TO THE OPPOSITE CORNER (A to B IN THE FIGURE) THAT ARE THE SHORTEST POSSIBLE DISTANCE, IN OTHER WORDS, WITH NO BACKTRACKING.

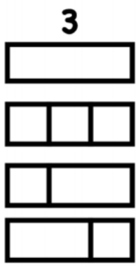
* A TYPICAL SHORTEST ROUTE IS SHOWN AS A BOLD PATH ON THE GRID IN THE FIGURE

TASK CARD #9

IMAGINE YOU HAVE RODS OF UNIT LENGTHS AS IN THE FOLLOWING DIAGRAM.

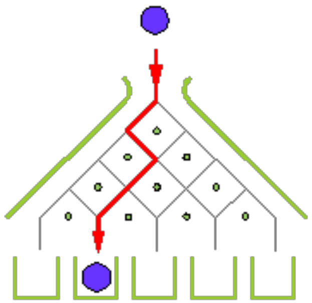


FIND OUT HOW MANY DIFFERENT ROD TRAINS CAN BE MADE FROM ANY LENGTH OF ROD. FOR EXAMPLE, YOU CAN MAKE THESE 4 TRAINS FOR THE 3 ROD.



TASK CARD #10

The pinball will be deflected either left or right with equal probability by the first nail. The result is that the pinball follows a random path, deflecting off one pin in each of the four rows of pins, and ending up in one of the cups at the bottom.



HOW MANY DIFFERENT PATHS ARE THERE THROUGH THE PINBALL MACHINE AND WHAT ARE THEY?

HOW MANY PATHS ARE THERE THAT END UP IN ANY GIVEN BIN?

WHAT IS THE PROBABILITY THAT THE PINBALL WILL END UP IN ANY GIVEN BIN?